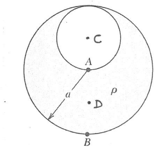
Electrodynamics
ISI B.Math
Final Exam : April 29, 2026

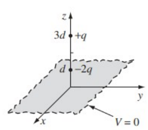
Total Marks: 75
Time : 3 hours
Answer all questions

1. (**Marks: 7 + 2 + 6**)



(a) A solid non-conducting sphere of radius a is filled with positive charge of uniform charge density ρ. Then a smaller sphere of radius $\frac{a}{2}$ is carved out as shown in the figure and left empty. What is the direction and magnitude of the electric field $\mathbf{E}$ at A? At B?

(b) Find $\nabla \cdot \mathbf{E}$ and $\nabla \times \mathbf{E}$ at the points C and D , where C and D are points in the interior of the empty region and the charge filled region respectively.



(c) Find the force on the charge $+q$ in the figure (The $x - y$ plane is a grounded conductor.)

2. (**Marks: 4 + 3 + 4 + 4 = 15**)

(a) Write down the full set of Maxwell's equations for the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ in the presence of a charge density ρ and current density $\mathbf{J}$ in differential form. Comment on the asymmetry between the magnetic and electric field in the equations involving the divergence of the fields.

(b) Derive the continuity equation relating the charge density ρ and current density $\mathbf{J}$ from Maxwell's equations. Explain how the continuity equation is an expression of the local conservation of charge.

(c) Show that a magnetic field $\mathbf{B}$ and electric field $\mathbf{E}$ that is a solution to Maxwell's equations can always be written as

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}$$

where ϕ is a scalar function and $\mathbf{A}$ is a vector field.

(d) Starting from source free Maxwell's equations in vacuum, show that each component of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ obeys a wave equation. Find the velocity of the wave in terms of ϵ_0 and μ_0.(Note that :$\nabla \times \nabla \times \mathbf{A} = -\nabla^2 \mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$)

3 . (**Marks: 2 + 3 + 5 + 5 = 15**)
Consider the following electric and magnetic fields whose real parts represent monochromatic plane waves propagating with velocity $c = k/\omega$.

$$\tilde{\mathbf{E}}(x,t) = \tilde{\mathbf{E}}_{\mathbf{0}} e^{i(kx - \omega t)}$$

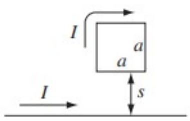
$$\tilde{\mathbf{B}}(x,t) = \tilde{\mathbf{B}}_{\mathbf{0}} e^{i(kx - \omega t)}$$

(i) What is the direction of propagation of the waves and what is the frequency ?
(ii) If the waves satisfy Maxwell's equations, show that the waves are transverse
(iii) If the waves satisfy Maxwell's equations show that

$$\tilde{\mathbf{B_0}} = \frac{k}{\omega}(\hat{\mathbf{x}} \times \tilde{\mathbf{E_0}})$$

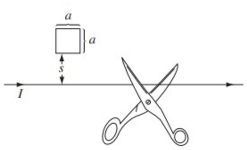
Hence, if the direction of polarization of the Electric field is $\hat{\mathbf{z}}$, what is the direction of polarization of the magnetic field?
iv) Find the time average (over a cycle) of the Poynting vector for such a wave. What does the Poynting vector represent physically ?

4. (**Marks: 7 + 8**)

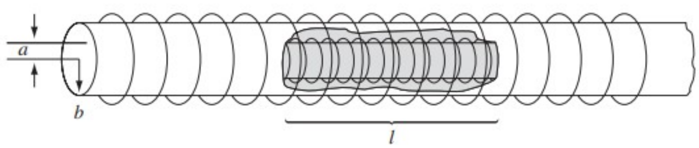


(a) Find the force on a square loop placed as shown in the figure near an infinite straight wire. Both the loop and wire carry a steady current I.
(b) Show that the magnetic vector potential $\mathbf{A}$ defined through $\mathbf{B} = \nabla \times \mathbf{A}$ is not unique and show that this non-uniqueness can be exploited to set $\nabla \cdot \mathbf{A} = 0$. For a uniform magnetic field it is given that $\mathbf{A}(\mathbf{r}) = -\frac{1}{2}(\mathbf{r} \times \mathbf{B})$.Show that it is indeed true that $\nabla \cdot \mathbf{A} = 0$

5. (**Marks: 7 + 8**)



A square loop of side a, resistance R lies a distance s from an infinite straight wire that carries a current I. Now someone cuts the wire, so that I drops to zero. In which direction does the induced current in the square loop flow, and what total charge passes a given point in the loop during the time this current flows ?



(b) A short solenoid (length l and radius a, with n_1 turns per unit length) lies on the axis of a very long solenoid(radius b, n_2 turns per unit length) as shown in the figure. Current I flows in the short solenoid. What is the flux through the short solenoid? Find the mutual inductance. [Recall that the magnitude of the magnetic field of an infinite solenoid is given by $\mu_0 n I$, where n is the number of turns per unit length and I is the current flowing through it.

Information you may or may not need:

$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$
$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$